Optimization of a Sustainable Inventory Model for Non-Instantaneous Deteriorating Items with Preservation Technology under Inflation and Renewable Energy

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Abstract
Due to global warming and increasing environmental issues sustainability becoming as an essential phenomenon in the operations management therefore industrialists is enforced to take sustainable practices such as renewable energy. On the other hand, governments are imposing carbon tax to reduce the emission due to the inventory management. A sustainable inventory model has been developed here for the decaying items, and the deterioration is assumed to be non-instantaneous. Preservation technology has been considered to diminish the decaying rate. The renewable energy and carbon emission tax has been addressed to reflect the sustainable aspects of the system, while the costing has been analysed under the inflationary environment to address the economic aspects. Demand rate has taken as price dependent. Mathematical model has been developed, and the total profit is maximized considering the price, preservation technology, renewable energy coefficient and cycle length as the decision variables.

Keywords: renewable energy; carbon tax; non-instantaneous deterioration; preservation technology; inflation; price dependent demand

1. Introduction

Incorporating sustainability into inventory management is essential due to the growing environmental concerns, resource constraints, and regulatory pressures faced by modern supply chains. Traditional inventory models often neglect the environmental impact of overstocking, waste generation, and inefficient resource use. However, as businesses strive to reduce their carbon footprint and support circular economy practices, it becomes critical to align inventory decisions with sustainability goals. Sustainable inventory management not only minimizes environmental harm through efficient resource utilization and waste reduction but also enhances long-term profitability, compliance, and brand reputation in an increasingly eco-conscious market. As environmental concerns grow and resources become scarcer, businesses are realizing that managing inventory isn't just about cutting costs anymore—it's also about making responsible, sustainable choices. Traditional inventory models often focus only on minimizing costs related to ordering, holding, or shortages, but they tend to overlook the environmental impact of these decisions. Excess production, waste, and high energy consumption all contribute to environmental degradation. That's why bringing sustainability into inventory management has become more important than ever.

A key part of this shift is the use of renewable energy. Inventory operations like manufacturing, storing goods in warehouses, and transportation typically consume a lot of energy—usually from fossil fuels. Switching to clean energy sources such as solar or wind power can significantly reduce the carbon footprint of these activities. It also opens up new ways to plan inventory—for instance, aligning production schedules with peak solar energy availability or designing eco-friendly storage systems. By adopting renewable energy, companies not only help protect the environment but also strengthen their long-term resilience and appeal to environmentally conscious consumers.

In addition to clean energy, government policies such as carbon taxes are driving companies to think differently about their environmental responsibilities. A carbon tax places a financial cost on greenhouse gas emissions, encouraging firms to reduce their reliance on polluting practices. For inventory systems, this means rethinking how often to produce, how much to store, and how far to transport goods—all with the goal of minimizing emissions-related costs. Integrating carbon tax considerations into inventory models helps businesses make more eco-friendly decisions while also preparing them to stay competitive in markets that increasingly value sustainability. In this way, carbon taxation acts not just as a penalty, but as a powerful incentive for greener, smarter inventory management.

While energy use and emissions are key environmental concerns, another important factor in sustainable inventory management is product deterioration. Many items—especially food, pharmaceuticals, chemicals, and fashion goods—have a limited shelf life. If not sold or used within a certain time frame, they lose value or become unusable, leading to waste, increased disposal costs, and environmental harm. This silent loss not only impacts profitability but also contradicts the principles of sustainability.

When deterioration is ignored in inventory decisions, it often results in overstocking, unnecessary production, and higher carbon emissions due to extra storage and transportation needs—all of which conflict with the goals of using renewable energy and minimizing environmental taxes like carbon levies. By incorporating deterioration into inventory models, businesses can make smarter decisions about order quantities, storage conditions, and replenishment frequency to reduce waste and environmental impact.

Sustainable inventory management, therefore, isn't just about what and how much to order— it's also about when to order and how long products can last without becoming a liability. Addressing deterioration not only cuts costs and waste but also supports broader goals of carbon reduction and resource efficiency, tying directly into the larger picture of sustainability that includes clean energy use and compliance with environmental regulations like carbon taxation.

Not all items deteriorate immediately after production or procurement—many go through a non-instantaneous deterioration process, where their quality or usability declines gradually over time. This is particularly true for products like fruits, dairy, cosmetics, medicines, and even electronic components. In such cases, items may remain usable for a certain period before they start losing value. This time lag provides a critical opportunity for more efficient and sustainable inventory planning. By accounting for non-instantaneous deterioration, businesses can better synchronize production, delivery, and sales to minimize losses. For instance, inventory models that consider gradual spoilage can help determine optimal order quantities and storage durations that reduce waste without compromising service levels. When aligned with strategies like renewable energy use and carbon cost minimization, such models enable companies to operate more responsibly and cost-effectively.

To further address the challenge of product deterioration—especially non-instantaneous deterioration—many businesses are turning to preservation technologies as part of their sustainable inventory strategy. These technologies, which include temperature-controlled storage, modified atmosphere packaging, natural preservatives, and smart monitoring systems, can significantly extend the usable life of products. By slowing down the deterioration process, they help reduce waste, minimize reordering frequency, and lower the environmental burden associated with frequent production and transportation. Incorporating preservation technology into inventory models enables more accurate planning of order quantities, storage durations, and replenishment cycles. For example, investing in cold chain systems for perishable goods can lead to fewer spoilage-related losses and reduce the need for overproduction—thereby conserving resources and lowering emissions. This complements broader sustainability efforts such as the use of renewable energy and the management of carbon taxes, as longer-lasting products reduce the intensity of energy consumption and carbon output per unit.

Another important factor that influences sustainable inventory decisions is inflation. Over time, the purchasing power of money decreases, affecting the costs of raw materials, production, storage, and transportation. Ignoring inflation in inventory planning can lead to underestimating future costs, resulting in inefficient budgeting and resource allocation.

In the context of sustainability, inflation not only impacts financial planning but also interacts with environmental strategies. For example, the rising cost of conventional energy may further justify the investment in renewable alternatives, while inflation-driven increases in packaging or storage costs can encourage leaner, more efficient inventory systems. Integrating inflation into inventory models helps firms prepare for long-term financial and environmental stability, ensuring that sustainability efforts remain economically viable over time.

Finally, an essential consideration in sustainable inventory modeling is the behavior of pricedependent demand. In many industries, customer demand responds directly to changes in price—lower prices may boost sales, while higher prices may suppress them. However, pricing decisions must be made carefully in the context of sustainability. Aggressively reducing prices to stimulate demand can lead to overproduction, excessive resource use, and higher emissions, ultimately conflicting with environmental goals.

When demand is influenced by price, it becomes even more critical to account for factors like product deterioration, especially non-instantaneous deterioration, to avoid generating unsellable surplus. Preservation technologies can help align supply with fluctuating demand by extending product life, while renewable energy integration can reduce the environmental cost of storing and handling larger inventories. At the same time, external factors like inflation and carbon taxes further complicate pricing and inventory decisions—rising costs due to inflation or penalties from carbon emissions can reduce profit margins, making sustainability-linked strategies not just environmentally sound, but economically essential.

By developing inventory models that incorporate price-sensitive demand alongside these sustainability factors, businesses can make more informed and balanced decisions—striving not only for profitability, but also for environmental responsibility and long-term resilience. This integrated approach marks a shift from traditional inventory thinking to a more holistic, future-focused strategy that aligns economic efficiency with ecological and social priorities.

2. Literature Review

Recent advancements in sustainable inventory management played a critical role of carbon tax policies in reducing emissions across operations management. Several researchers have integrated carbon taxation into mathematical models to assess its impact on inventory decisions. Giri and Roy (2016) expanded on this by incorporating controllable lead time, finding that while reducing lead time may decrease selling prices, it can simultaneously enhance overall system profits. Yılmaz Balaman and Selim (2016) added a broader supply chain dimension by presenting a fuzzy model that integrates biomass inventory and energy production, within renewable district heating systems. Sustainable inventory management becomes increasingly complex when deterioration particularly non-instantaneous and controllable deterioration is incorporated into system design. Tsao (2016) have further illustrated the impact of preservation effort, pricing, and marketing on deterioration-driven inventory models, suggesting that deterioration control is not merely a storage issue but a strategic decision influenced by location, cost, and demand response. Mishra and Mishra (2020) explored a model that integrates environmental emission control and preservation technologies to handle deterioration sustainably, showing that joint optimization of investment in preservation and replenishment strategies can significantly enhance profits while reducing environmental impacts.

Rout et al. (2021) proposed bi-objective models that optimize cost and emissions by jointly considering deterioration, transportation, and rework decisions. A vendor-managed inventory system is developed by Guchhait and Sarkar (2021) with blockchain, incorporating carbon tax to control emissions in decentralized environments. Choudhury et al. (2021) developed a model for deteriorating items with quality loss and quantity loss with expiration date. Preservation technology has emerged as a vital strategy in sustainable inventory management, particularly for controlling the degradation of perishable items and enhancing system profitability.

Numerous researchers have demonstrated that optimal investment in preservation technologies can significantly reduce deterioration rates while supporting demand and environmental objectives. In the same year, Priyamvada et al. (2021) revisited an EOQ model incorporating preservation investment under price and stock-dependent demand, showing its potential in jointly optimizing cycle length and profits. Mashud and Sarkar (2021) established that combining preservation strategies with trade-credit policies creates a synergy that enhances sustainability and marketing performance. Collectively, these studies affirm that integrating preservation technology into inventory models is not only a deterioration-control tactic but also a strategic lever for achieving cost-effective and sustainable supply chain operations. Price-dependent demand plays a pivotal role in sustainable inventory management, especially when integrated with modern supply chain strategies and deterioration dynamics. Pan et al. (2021) took a broader perspective by incorporating price, advertising, and carbon emission reduction technologies in a multi-stage supply chain, optimizing profit while supporting sustainable production and inventory decisions. Naseri et al. (2021) addressed price-dependent stochastic demand within a hybrid production system including remanufacturing and refurbishing, highlighting the critical interaction between pricing, customer returns, and recovery costs. In models involving deteriorating items, Nath and Sen (2021) introduced a two-warehouse structure with time and price-dependent demand, allowing for complete backlogging and demonstrating how pricing strategies affect storage decisions and cost minimization. Collectively, these studies underscore the necessity of integrating price sensitivity into inventory models to support both economic efficiency and sustainability in dynamic supply chain environments.

Similarly, Sundararajan et al. (2022) examined the effect of inflation and delayed payment in EOQ models with non-instantaneous deterioration and partial backlogging, emphasizing the importance of financial parameters in sustainable decision-making. A model is explored by Dominioni et al. (2022) that implements multiple carbon policies carbon tax, cap-and-trade, and offset demonstrating significant emission reductions alongside uncertain production parameters. These studies collectively advance the integration of deterioration dynamics and reliability factors into sustainable inventory decision-making. Overall, carbon taxation has proven to be a powerful regulatory mechanism, encouraging firms to internalize environmental costs and adopt more sustainable inventory strategies.

Other studies focus on renewable energy in inventory management. The integration of renewable energy sources into inventory and supply chain systems has emerged as a critical component of sustainable inventory management. Lee et al. (2022) developed competitive bidding strategies for real-time inventory management in electricity markets powered by renewables. Feng and Menezes (2022) proposed an integrated make-to-stock inventory model incorporating renewable energy, offering structural policies for energy efficiency in production systems. Meanwhile, Xiong et al. (2022) addressed price fluctuations in renewable energy technology supply chains and proposed contract designs to maintain inventory efficiency despite market volatility. These studies collectively highlight how energy-aware inventory models not only improve operational efficiency but also help achieve environmental goals.

Further research has delved into system-level implementations of renewable-integrated inventory systems. Ijuin et al. (2022) introduced a solar energy-based demand-to-supply inventory management method, enhancing stability in renewable-dependent systems. Patel et al. (2022) developed conceptual model that may be used to maintain a sustainable supply chain with electric vehicles in such a way that caters to both environmental concerns and human requirements.. These advancements underscore that sustainable inventory management increasingly depends on the successful coordination of renewable energy technologies with inventory control, logistics, and strategic planning laying the groundwork for climate-resilient and energy-efficient operations. Mahapatra et al. (2022) introduced a model to study employs preservation technologies under uncertain demand to frame a continuous review inventory model with full backordering and the influence of promotional efforts.

3. Assumption and Notations
3.1 Assumptions
- An Economic Order Quantity (EOQ) model is formulated for deteriorating items. The deterioration is considered to be non-instantaneous, where the items start to decay after a certain time.
- To mitigate deterioration, preservation technology is incorporated into the model. The cost of preservation technology is considered to be $\mu e^{\gamma\xi}$ while ξ is the reduction in the deterioration due to the preservation technology.
- The demand rate is assumed to be sensitive to the selling price and is taken as $\alpha p^{-\beta}$, where is the p price and α and $\beta > 0$.
- The model incorporates the use of renewable energy, which helps in lowering the electricity costs associated with inventory holding. Consequently, this reduces the per unit holding cost per unit time. The cost of renewable energy is taken as $C_R\zeta$, where ζ is the reduction parameter
- Carbon emissions generated during the inventory process are considered, and a carbon tax is imposed to motivate firms to minimize their environmental pollution.
- The entire model is constructed within an inflationary economic framework.

3.2 Notations
The following notations are used in the model.

α	Potential demand rate
β	Price sensitive demand parameter
θ	Deterioration rate
ξ	Preservation technology parameter
r	Inflation rate
p	Price
O	Ordering cost
h	Holding cost
ζ	Renewable energy parameter
C_d	Deterioration cost
μ	Preservation technology cost scale parameter
γ	Preservation technology cost shape parameter
C_R	Renewable energy parameter
E_0	Constant carbon emission
c_0	Constant carbon tax
E_h	Variable carbon emission
c_1	Variable carbon tax

4. Mathematical Modelling
The inventory model considers a cycle of fixed length T, where the inventory is initially replenished at time $t = 0$. During the initial phase $[0, t_1]$, the inventory level decreases solely due to customer demand, as the items do not deteriorate in this period. After time t_1, deterioration begins, and the inventory level declines due to both demand and deterioration simultaneously. A differential equation-based approach is used to represent the inventory behaviour in each phase, ensuring a continuous and realistic depiction of stock depletion over time. The model integrates the effects of price-sensitive demand, deterioration, and preservation technology to optimize the total cost.

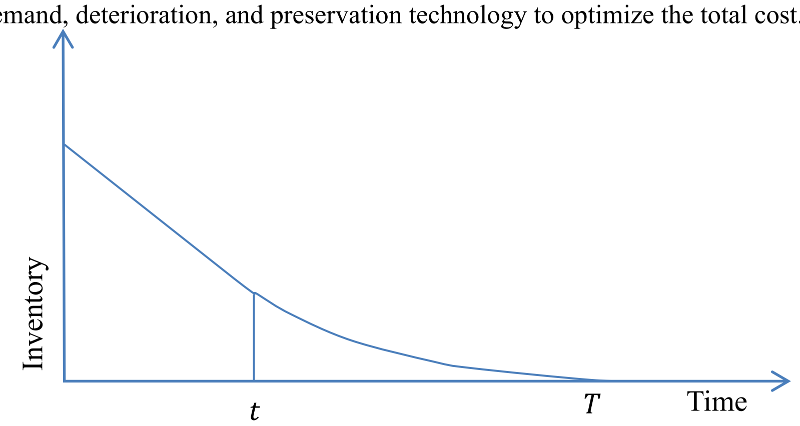

Figure 1: Inventory level of the EOQ model

The inventory level is governed by the following differential equations

$$\frac{dQ_1(t)}{dt} = -\alpha p^{-\beta} \qquad Q_1(t_1) = Q_2(t_1) \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dQ_2(t)}{dt} + (\theta - \xi)Q_2(t) = -\alpha p^{-\beta} \qquad Q_2(T) = 0 \qquad t_1 \le t \le T \tag{2}$$

Solution of these equations is

$$Q_1(t) = \frac{p^{-\beta}\alpha\left(e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)(t_1-t) - 1\right)}{(\theta-\xi)} \qquad 0 \le t \le t_1 \tag{3}$$

$$Q_2(t) = \frac{\left(e^{(T-t)(\theta-\xi)} - 1\right)p^{-\beta}\alpha}{(\theta-\xi)} \qquad t_1 \le t \le T \tag{4}$$

Order quantity

$$Q = Q_1(0) = \frac{p^{-\beta}\alpha\left(-1 + e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)t_1\right)}{(\theta-\xi)} \tag{5}$$

Total inventory under the inflationary environment

$$\int_0^T \left(Q(t)\right)e^{rt}\,dt = \int_0^{t1} Q_1(t)e^{rt}\,dt + \int_{t1}^T Q_2(t)e^{rt}\,dt$$

$$= \int_0^{t1} \left(\frac{p^{-\beta}\alpha\left(e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)(t_1-t) - 1\right)}{(\theta-\xi)}\right)e^{rt}\,dt + \int_{t1}^T \left(\frac{\left(e^{(T-t)(\theta-\xi)} - 1\right)p^{-\beta}\alpha}{(\theta-\xi)}\right)e^{rt}\,dt$$

$$= \frac{p^{-\beta}\alpha}{r^2(\theta-\xi)(\theta-\xi-r)}\left\{\left((e^{rt_1}-1)(\theta-\xi)^2 + r^2\left(e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)t_1 - 1\right) - r(\theta-\xi)\left(e^{rT} + e^{(\theta-\xi)(T-t_1)} + e^{rt_1} - e^{(\theta-\xi)(T-t_1)+rt_1} + (\theta-\xi)t_1 - 2\right)\right)\right\} \tag{6}$$

1. **Ordering cost**

$$OC = 0$$

2. **Holding cost**

$$HC = h(1-\zeta)\left(\int_0^{t1} Q_1(t)e^{rt}\,dt + \int_{t1}^T Q_2(t)e^{rt}\,dt\right)$$

$$= h(1-\zeta)\left\{\int_0^{t1}\left(\frac{p^{-\beta}\alpha\left(e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)(t_1-t) - 1\right)}{(\theta-\xi)}\right)e^{rt}\,dt + \int_{t1}^T\left(\frac{\left(e^{(T-t)(\theta-\xi)} - 1\right)p^{-\beta}\alpha}{(\theta-\xi)}\right)e^{rt}\,dt\right\}$$

$$= \frac{p^{-\beta}\alpha h(1-\zeta)}{r^2(\theta-\xi)(\theta-\xi-r)}\left\{\left((-1+e^{rt_1})(\theta-\xi)^2 + r^2\left(-1 + e^{(\theta-\xi)(T-t_1)} + (\theta-\xi)t_1\right) - r(\theta-\xi)\left(-2 + e^{rT} + e^{(\theta-\xi)(T-t_1)} + e^{rt_1} - e^{(\theta-\xi)(T-t_1)+rt_1} + (\theta-\xi)t_1\right)\right)\right\}$$

3. Deterioration cost

$$DC = C_d(\theta-\xi)\left(\int_0^{t1} Q_1(t)e^{rt}\,dt + \int_{t1}^{T} Q_2(t)e^{rt}\,dt\right)$$

$$= C_d(\theta-\xi)\left\{\int_0^{t1}\left(\frac{p^{-\beta}\alpha\left(e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)(t_1-t)-1\right)}{(\theta-\xi)}\right)e^{rt}\,dt + \int_{t1}^{T}\left(\frac{\left(e^{(T-t)(\theta-\xi)}-1\right)p^{-\beta}\alpha}{(\theta-\xi)}\right)e^{rt}\,dt\right\}$$

$$= \frac{p^{-\beta}\alpha C_d(\theta-\xi)}{r^2(\theta-\xi)(\theta-\xi-r)}\left\{\left((-1+e^{rt_1})(\theta-\xi)^2 + r^2\left(-1+e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)t_1\right) - r(\theta-\xi)\left(-2+e^{rT}+e^{(\theta-\xi)(T-t_1)}+e^{rt_1}-e^{(\theta-\xi)(T-t_1)+rt_1} + (\theta-\xi)t_1\right)\right)\right\}$$

4. Preservation technology

$$PTC = \mu e^{\gamma\xi}$$

5. Renewable energy cost

$$REC = C_R\zeta$$

6. Carbon emission cost

$$CEC = \frac{E_0c_0}{T} + E_hc_1\left(\int_0^{t1} Q_1(t)e^{rt}\,dt + \int_{t1}^{T} Q_2(t)e^{rt}\,dt\right)$$

$$= \frac{E_0c_0}{T} + E_hc_1\left(\left\{\int_0^{t1}\left(\frac{p^{-\beta}\alpha\left(e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)(t_1-t)-1\right)}{(\theta-\xi)}\right)e^{rt}\,dt + \int_{t1}^{T}\left(\frac{\left(e^{(T-t)(\theta-\xi)}-1\right)p^{-\beta}\alpha}{(\theta-\xi)}\right)e^{rt}\,dt\right\}\right)$$

$$= \frac{E_0c_0}{T} + E_hc_1\left(\frac{p^{-\beta}\alpha}{r^2(\theta-\xi)(\theta-\xi-r)}\left\{\left((-1+e^{rt_1})(\theta-\xi)^2 + r^2\left(-1+e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)t_1\right) - r(\theta-\xi)\left(-2+e^{rT}+e^{(\theta-\xi)(T-t_1)}+e^{rt_1}-e^{(\theta-\xi)(T-t_1)+rt_1} + (\theta-\xi)t_1\right)\right)\right\}\right)$$

7. Revenue generated

$$P = p\int_0^{T}\left(\alpha p^{-\beta}\right)e^{rt}\,dt = \frac{\alpha p^{1-\beta}(e^{rT}-1)}{r}$$

$$\text{Total Cost} = \frac{1}{T}\big(\text{Revenue generated} - (\text{Ordering cost} + \text{Holding cost} + \text{Deterioration cost} + \text{Preservation technology} + \text{Renewable energy cost} + \text{Carbon emission cost})\big)$$

$$
\begin{aligned}
TP = \frac{1}{T}\Bigg[& \frac{\alpha p^{1-\beta}(e^{rT}-1)}{r} \\
& -\Bigg\{O \\
& + \frac{p^{-\beta}\alpha h(1-\zeta)}{r^2(\theta-\xi)(\theta-\xi-r)}\Big\{\Big((-1+e^{rt_1})(\theta-\xi)^2 \\
& + r^2\big(-1+e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)t_1\big) \\
& - r(\theta-\xi)\big(-2+e^{rT}+e^{(\theta-\xi)(T-t_1)}+e^{rt_1}-e^{(\theta-\xi)(T-t_1)+rt_1} \\
& + (\theta-\xi)t_1\big)\Big)\Big\} \\
& + \frac{p^{-\beta}\alpha C_d(\theta-\xi)}{r^2(\theta-\xi)(\theta-\xi-r)}\Big\{\Big((-1+e^{rt_1})(\theta-\xi)^2 \\
& + r^2\big(-1+e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)t_1\big) \\
& - r(\theta-\xi)\big(-2+e^{rT}+e^{(\theta-\xi)(T-t_1)}+e^{rt_1}-e^{(\theta-\xi)(T-t_1)+rt_1} \\
& + (\theta-\xi)t_1\big)\Big)\Big\} + \mu e^{\gamma\xi} + C_R\zeta + \frac{E_0 c_0}{T} \\
& + E_h c_1\Bigg(\frac{p^{-\beta}\alpha}{r^2(\theta-\xi)(\theta-\xi-r)}\Big\{\Big((-1+e^{rt_1})(\theta-\xi)^2 \\
& + r^2\big(-1+e^{(\theta-\xi)(T-t_1)}+(\theta-\xi)t_1\big) \\
& - r(\theta-\xi)\big(-2+e^{rT}+e^{(\theta-\xi)(T-t_1)}+e^{rt_1}-e^{(\theta-\xi)(T-t_1)+rt_1} \\
& + (\theta-\xi)t_1\big)\Big)\Big\}\Bigg)\Bigg\}\Bigg] \\
= \frac{1}{T}\Bigg[& \frac{(e^{rT}-1)p^{1-\beta}\alpha}{r} - O - e^{\gamma\xi}\mu - \frac{ec_0}{T} - \zeta C_R \\
& - \frac{1}{r(\theta-\xi)}p^{-\beta}\alpha\Bigg\{\frac{1}{(r-1)}\Big(-e^{(\theta-\xi)(T-t_1)+t_1 r}+e^{(\theta-\xi)(T-t_1)+rt_1}r \\
& + (-1+r)(\theta-\xi) + e^{t_1}r(1-\theta+\xi) - e^{rt_1}(r-\theta+\xi)\Big) \\
& + \frac{-e^{(\theta-\xi)(T-t_1)+rt_1}r + e^{rT}(\theta-\xi) + e^{rt_1}(r-\theta+\xi)}{r-\theta+\xi}\Bigg\}\big(h(1-\zeta) \\
& + (\theta-\xi)C_d + c_1 e_h\big)\Bigg]
\end{aligned} \tag{7}
$$

The objective function given above is a function of the decision variables p and T. To find the optimum value of the total profit one need to equate the derivative of the total cost equation (7) to zero $i.e.$

$\frac{dTP}{dT} = 0$ and $\frac{dTP}{dp} = 0$

5. **Numerical analysis**

To demonstrate the applicability of the proposed sustainable inventory model, a numerical analysis is carried out by assigning suitable values to the system parameters. The aim is to observe the behaviour of key performance indicators such as total profit, optimal price, and cycle length under different conditions. The chosen parametric values reflect realistic industrial settings and help analyse the sensitivity of the model to various decision variables.

$O = 1000,\quad r = 0.1,\quad p = 100,\quad e_h = 1,\quad \alpha = 1000,\quad \beta = 0.5,$
$\gamma = 100,\quad \xi = 0.005\quad ,\ \mu = 1000,\quad c_0 = 5000,\quad e_0 = 5,$
$\zeta = 0.5,\quad C_R = 2000\quad ,\quad \theta = 0.1,\quad h = 5,\quad C_d = 7.5,$
$c_1 = 1.5,\quad t_1 = 2.5$

Table 1 presents the optimal values of the decision variables and the corresponding objective function (total profit). At an optimal price p = 16.38 and cycle length T = 8.5066, the maximum total profit (TP) achieved is 7131.87, confirming the effectiveness of the proposed model in maximizing profitability under given conditions.

Table 1: **optimal results of the decision variables and the objective function**

p	T	TP
16.38	8.5066	7131.87

Figure 2 illustrates the convex nature of the total profit function with respect to the price and cycle length. The surface plot shows a clear peak, indicating the presence of a unique global maximum. This validates the convexity of the objective function and ensures the reliability of the optimization results.

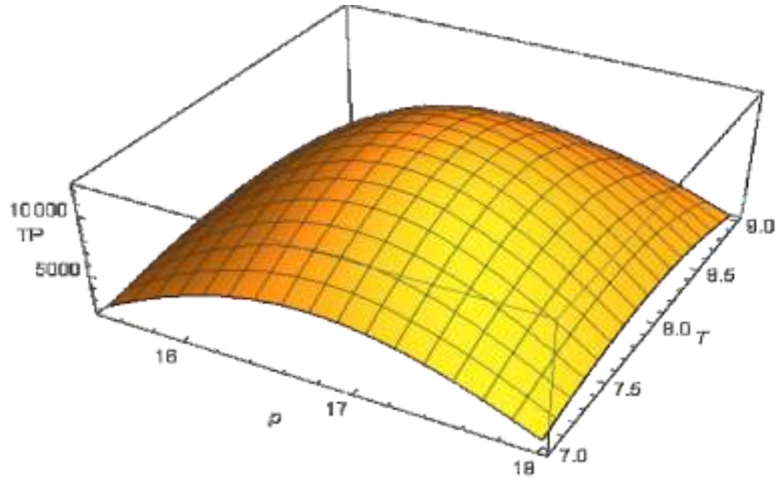

Figure 2: convexity of the profit function

6. Sensitivity Analysis

Table 2 summarizes the sensitivity analysis of the decision variables and total profit with respect to changes in key system parameters such as demand rate, price sensitivity, preservation investment, and renewable energy coefficient. The results provide insight into how small changes in these parameters affect optimal decisions and profitability, offering valuable guidance for managerial decision-making.

Table 2: **sensitivity analysis of the decision variables and the objective function with respect to the different parameters**

Parameter	Parametric value	p	T	TP
α	800	16.59	9.07952	5558.06
	900	16.40	8.78121	6343.04
	1000	16.38	8.5066	7131.87
	1100	15.76	8.24968	7924.2
	1200	15.25	8.00586	8719.77
β	0.3	8.82	4.82765	19458.1
	0.4	13.26	7.13146	11809.8
	0.5	16.38	8.5066	7131.87
	0.6	17.54	9.66302	4241.4
	0.7	19.88	10.7712	2456.87
ξ	0.003	15.42	8.10294	7102.28
	0.004	15.56	8.29749	7116.99
	0.005	16.38	8.5066	7131.87
	0.006	16.66	8.73136	7146.99
	0.007	16.55	8.97283	7162.44
ζ	0.3	11.12	5.71725	5858.19
	0.4	12.90	6.76495	6424.25
	0.5	16.38	8.5066	7131.87
	0.6	21.08	11.4216	8103.64
	0.7	29.64	15.7226	9604.48

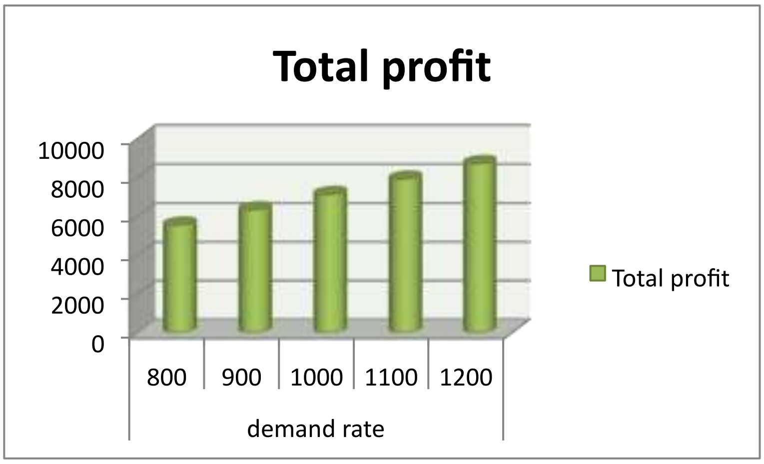

Figure 3: Sensitivity Analysis of total profit with respect to the demand rate

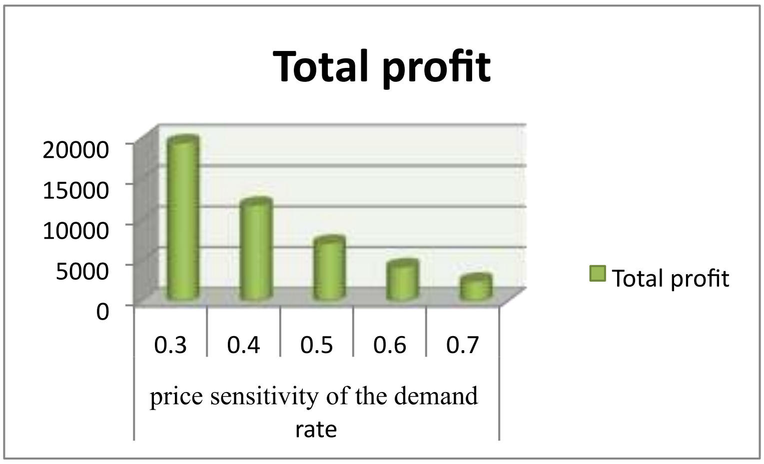

Figure 4: Sensitivity Analysis of total profit with respect to the price sensitivity of the demand rate

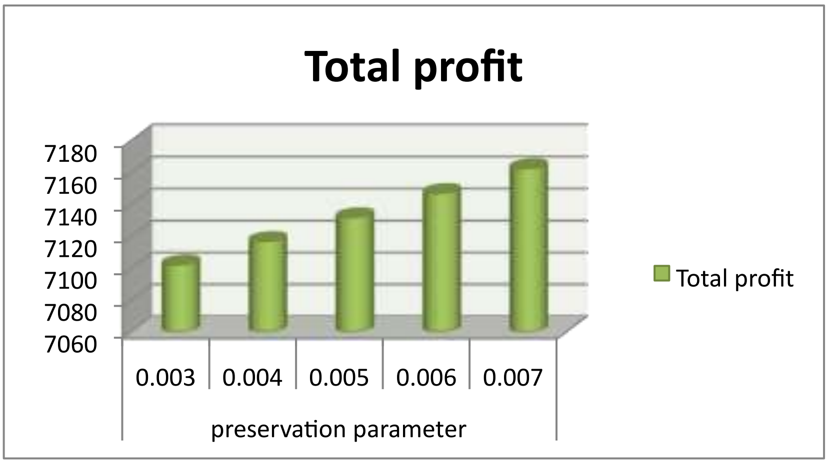

Figure 5: Sensitivity Analysis of total profit with respect to the preservation parameter

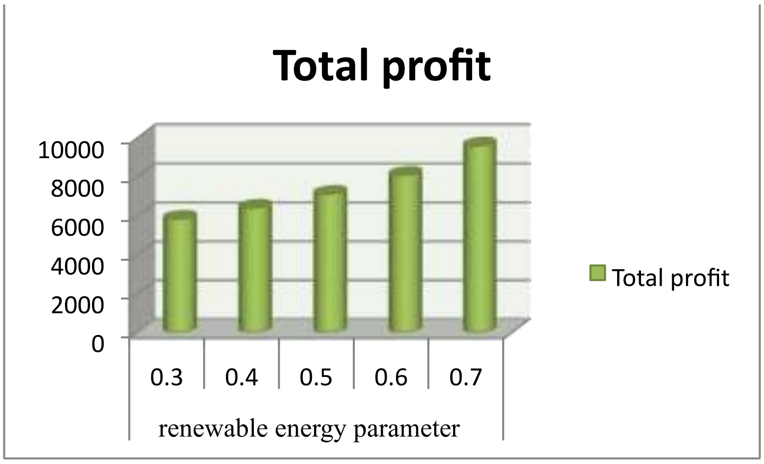

Figure 6: Sensitivity Analysis of total profit with respect to the renewable energy parameter

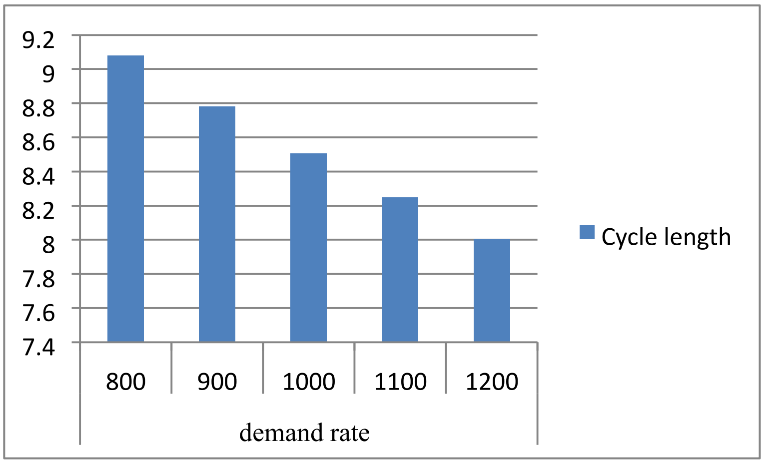

Figure 7: Sensitivity Analysis of Cycle length with respect to the demand rate

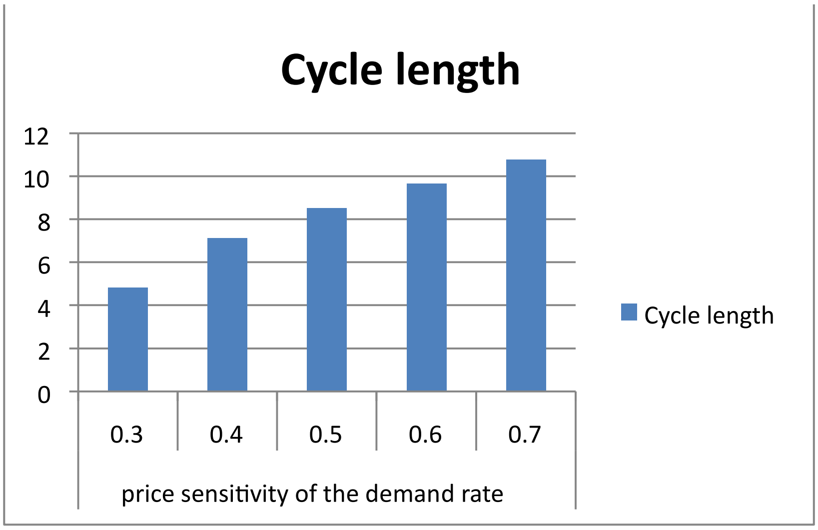

Figure 8: Sensitivity Analysis of Cycle length with respect to the price sensitivity of the demand rate

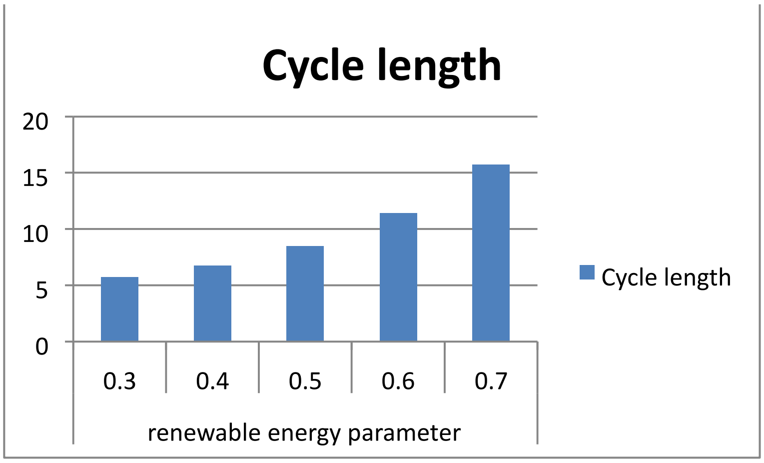

Figure 9: Sensitivity Analysis of Cycle length with respect to the renewable energy parameter

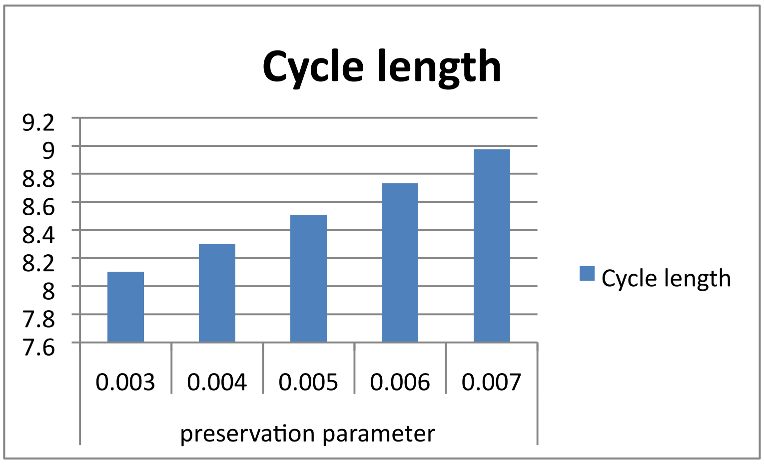

Figure 10: Sensitivity Analysis of Cycle length with respect to the preservation parameter

Figure 11: Sensitivity Analysis of price with respect to the demand rate

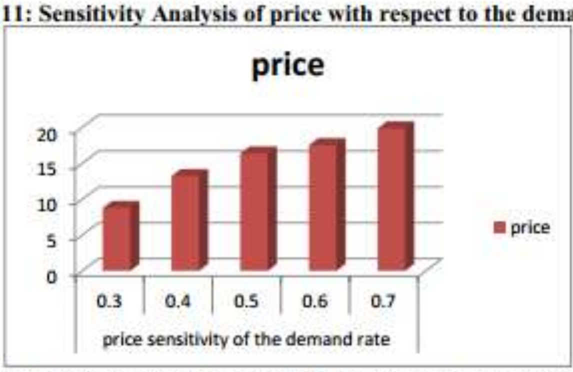

Figure 12: Sensitivity Analysis of price with respect to the price sensitivity of the demand rate

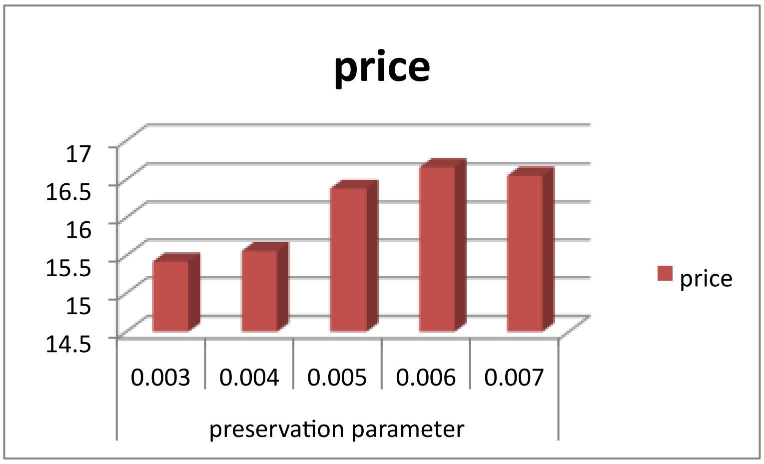

Figure 13: Sensitivity Analysis of price with respect to the preservation parameter

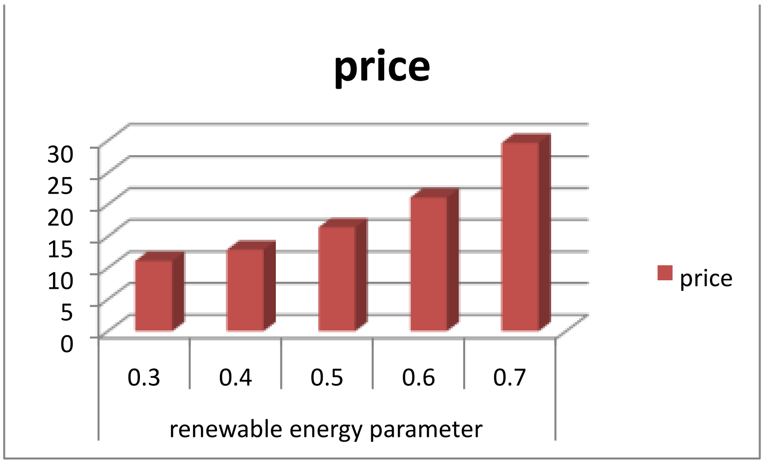

Figure 14: Sensitivity Analysis of price with respect to the renewable energy parameter

Observations

- Figure 3 shows that as the demand rate increases, the total profit rises steadily, indicating a positive linear relationship between demand and profitability.
- In Figure 4, it is observed that an increase in price sensitivity leads to a sharp decline in total profit, suggesting that the demand is highly elastic—small changes in price significantly reduce sales and revenue.
- Figure 5 highlights that with an increase in the preservation parameter, the total profit increases slightly, indicating a minor positive impact of preservation technology in mitigating deterioration-related losses.
- Figure 6 illustrates that as the renewable energy parameter increases, total profit rises significantly. This suggests that incorporating more renewable energy either reduces operational costs or improves efficiency, thereby boosting overall profit.
- Figure 7 demonstrates that with higher demand rates, the optimal price decreases, reflecting a demand-driven pricing approach aimed at attracting more customers through lower prices.
- In contrast, Figure 8 reveals that as price sensitivity increases, the optimal price also increases significantly. This may be attributed to the need to maximize per-unit revenue when higher sensitivity results in lower quantities sold.
- Figure 9 shows a slight increase in optimal price with rising preservation parameters. This indicates that better preservation improves product quality or reduces losses, justifying a moderate price premium.
- Figure 10 reveals a sharp increase in optimal price with greater use of renewable energy, possibly due to sustainability branding or consumer preference for green products, which supports a premium pricing strategy.
- Figure 11 indicates that as the demand rate increases, the cycle length decreases, suggesting that higher demand leads to faster inventory depletion and therefore shorter replenishment cycles.
- Figure 12 shows that with rising price sensitivity, the cycle length increases, likely because fewer units are sold at higher prices, resulting in slower inventory turnover.
- Figure 13 illustrates that as the preservation parameter increases; the cycle length also increases slightly, as improved preservation allows products to be held longer.

- Lastly, Figure 14 demonstrates that with a higher renewable energy parameter, the cycle length increases significantly. This trend implies that greater use of renewable energy promotes cost efficiency or sustainable production, enabling longer and less frequent replenishment cycles.

7. **Conclusion**

This article presents a sustainable inventory model for non-instantaneous deteriorating items, considering key environmental and economic factors such as preservation technology, renewable energy usage, carbon emission tax, and inflation. The demand is modelled as price-dependent, and the objective is to maximize total profit by optimizing four decision variables: selling price, preservation investment, renewable energy parameter, and replenishment cycle length. The model is designed to reflect real-world operational challenges faced by industries striving for sustainability while maintaining profitability in inflationary settings.

The results from the sensitivity analysis offer meaningful managerial insights. It is observed that an increase in demand rate leads to a steady rise in total profit, while higher price sensitivity drastically reduces profitability, indicating highly elastic market behaviour. Preservation technology yields a marginal increase in both profit and optimal price, demonstrating its moderate but positive influence on inventory sustainability. On the other hand, renewable energy adoption significantly enhances both total profit and optimal price, supporting the idea that green practices can lead to higher operational efficiency and value added branding. Cycle length tends to shorten with higher demand and lengthen with increased price sensitivity, preservation, and renewable energy, reflecting adjustments in replenishment frequency in response to sustainability investments.

For future research, the model can be extended to incorporate stochastic demand, time varying deterioration rates, or probabilistic lead times to enhance its applicability in uncertain environments. Multi-item, multi-echelon, or supply chain-level models can be developed to capture broader logistics dynamics. The inclusion of carbon credit trading systems, green subsidies, or inventory-dependent emissions can enrich the sustainability perspective. Additionally, integrating machine learning or AI-driven demand forecasting techniques could further improve the model's responsiveness and decision-making accuracy in a real-time data driven context.

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