



On Certain New Generalised Closed set in Ideal Nano Topological Spaces

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Abstract

The purpose of this paper is to introduce some new classes of sets namely $\bar{\rho}gI$ - Closed set and $\bar{\rho}gI$ -Open sets in Ideal Nano topological space and investigate some basic topological properties and its characterizations. Also the relationship between these sets with some of the known closed sets are studied and analyzed.

Keywords: $\bar{\rho}gI$ - Closed set, $\bar{\rho}gI$ - Open set.

Introduction

Topological space is a set endowed with a structure called a topology. In 1930 we introduce the concept of Ideals in topological space. In 2013, Lellis Thivagar and Carmel Richard [8] established the field of nano topological spaces which was defined in terms of approximation and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. K. Bhuvaneswari et al. [6] introduced and studied the concept of nano generalised closed sets in nano topological spaces. [18] J. Arul Jesti and J. Joycy Renuka introduced the concept of NIg semi*-closed sets in Nano Ideal topological space.

In this paper we introduce a new type of $\bar{\rho}gI$ -closed set and $\bar{\rho}gI$ - open set in Ideal Nano topological space and investigated the relationships between these sets in Nano topological spaces and Ideal Nano topological spaces. Characterizations and properties of $\bar{\rho}gI$ - closed set and $\bar{\rho}gI$ - open set are also studied.

1. Preliminaries

Definition 1.1 [8]

Let U be the universe, R be an equivalence relation on U and

$\tau R(X) = \{U, \phi, LR(X), UR(X), BR(X)\}$, where $X \subseteq U$ and $\tau R(X)$ satisfies the following axioms

i) U and $\phi \in \tau R(X)$.

ii) The union of the elements of any sub-collection of $\tau R(X)$ is in $\tau R(X)$.

iii) The intersection of the elements of any finite sub collection of $\tau R(X)$ is in $\tau R(X)$. Therefore, $\tau R(X)$ as the nano topological space. The elements of $\tau R(X)$ are called nano open sets(briefly n-open sets). The complement of a nano open set is called a nano closed set(briefly n-closed set)

Definition 1. 2[18] A nano topological space (U, N) with an ideal I on U is called an ideal nano topological space or nano ideal topological space and is denoted by (U, I, N) .

Definition 1. 3[16]

Let (U, N, I) be a nano ideal topological space U , where $N = \tau R(X)$ and $(\cdot)_n^*$ be a set operator from $P(U) \rightarrow P(U)$.

$(P(U)$ is the set of all subsets of U). For a subset $H \subset U$, $H_n^*(I, N) = \{x \in U: G_n \cap H \notin I, \text{ for every}$

$G_n \in G_n(x)\}$, where $G_n(x) = \{G_n \mid x \in G_n, G_n \in N\}$ is called the nano local function (briefly, n-local function) of H with respect to I and N .

We simply write H_n^* for $H_n^*(I, N)$. The set operator n-cl is called a nano*-closure and is defined as $n-cl^*(H) = H \cup H_n^*$ for $H \subseteq X$.

Definition 1.5 A subset A of a nano ideal topological space (U, N, I) is said to be

i) NIg semi*-closed if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is nano semi*-open [18]

ii) NI_g -closed if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is nano open [5]

iii) NI_g -closed if $(A^*)^N \subseteq V$ whenever $A \subseteq V$ and V is nano semi-open [13]

iv) $NI_{g\alpha}$ -closed if $N\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and V is NI_g -open [12]

v) $NI_{g\alpha}$ -closed if $NI\alpha cl(A) \subseteq V$ whenever $A \subseteq V$, V is nano open [5]

vi) $*^N$ -closed if $(A)^{*N} \subseteq A$ [14]

Theorem 1.6 [18]

1) If (U, N, I) is a nano topological space with an ideal I and $A \subseteq A_n^*$, then $A_n^* = n\text{-cl}(A_n^*) = n\text{-cl}(A)$.

2) Every N_g closed set is NIg semi*-closed set.

3) Every nano semi open set is NIg semi*-open set.

Definition 1.7 [15] A subset A of a nano ideal topological space (U, N, I) is n^* dense itself (resp. n^* -perfect and n^* -closed) if $A \subseteq A_n^*$ (resp. $A = A_n^*$, $A_n^* \subseteq A$).

2. On $\bar{\rho}gI$ – Closed set in Ideal Nano Topological Space

Definition 2.1

A subset H of an Ideal Nano topological space (U, I, N) is said to be $\bar{\rho}gI$ -closed if $H_n^* \subseteq V$ whenever $H \subseteq V$ and V is NIg semi* open.

Example 2.2 Let $U = \{a, b, c, d\}$, $X = \{a, b\} \subset U$ and $I = \{\emptyset, \{a\}\}$.

$\tau_R(X) = \{U, \emptyset, \{a\}, \{a, c, d\}, \{c, d\}\}$,

$\bar{\rho}gIcl(U, I, N) = \{U, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.

Theorem 2.3 Every n^* -closed set is $\bar{\rho}gI$ – Closed but not conversely.

Proof : Let $H \subseteq X$ and H be n^* -closed set. Assume $H \subseteq V$ and V is NIg semi*-open. Since H is n^* -closed, $H_n^* \subseteq H$ also $H_n^* \subseteq H \subseteq V$. Therefore $H_n^* \subseteq V$

Hence H is $\bar{\rho}gI$ – Closed.

Example 2.4 Let $U = \{a, b, c, d\}$, $X = \{a, c\} \subset U$ and $I = \{\emptyset, \{b, c\}\}$.

$\bar{\rho}gIcl(U, I, N) = \{U, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.

n^* -closed set = $\{\emptyset, U, \{a, b\}\}$. The set $\{b, d\}$ is $\bar{\rho}gI$ – closed but not n^* -closed.

Theorem 2.5 Every N_g -closed set is $\bar{\rho}gI$ - closed.

Proof : Suppose H is N_g -closed. Let V be any nano-open set containing H .

By theorem 1.6(2), V is NIg semi*-open set Containing H . Also H is N_g -closed

we have $ncl(H) \subseteq V$. By 1.6(1), $H_n^* = ncl(H_n^*) = ncl(H) \Rightarrow H_n^* = ncl(H) \subseteq V$

$\Rightarrow H_n^* \subseteq V$. Thus H is $\bar{\rho}gI$ -closed

Remark: The converse of the above theorem need not be true as given in the following example. In Example 2.2 and

$N_gcl(U, I, N) = \{U, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.

The set $\{a\}$ is $\bar{\rho}gI$ -closed set but not N_g -closed set.

Theorem 2.6 Every $NI_{\hat{g}}$ -closed set is $\bar{\rho}gI$ -closed but not reversely.

Proof : Given H is $NI_{\hat{g}}$ -closed and $H \subseteq V$ where V is nano semi open.

Since by theorem[1.6(3)], V is NIg semi*open in H .

Therefore $H_n^* \subseteq V$. Hence H is $\bar{\rho}gI$ -closed.

Example 2.7 Let $U = \{a, b, c, d\}$, $X = \{c\} \subset U$ and $I = \{\emptyset, \{c\}\}$.

$\bar{\rho}gI$ -closed set = $\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

$NI_{\hat{g}}cl(U, I, N) = \{U, \emptyset, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$.

Clearly the set $\{a, c, d\}$ is $\bar{\rho}gI$ -closed but it is not $NI_{\hat{g}}$ -closed.

Theorem 2.8 Every $NI_{\hat{g}\alpha}$ -closed set is $\bar{\rho}gI$ -closed but not conversely.

Proof: Let H be $NI_{\hat{g}\alpha}$ -closed. Since every $NI_{\hat{g}\alpha}$ -closed set is $NI_{\hat{g}}$ -closed. Therefore $N\alpha cl(H) \subseteq$

$Ncl(H) \subseteq V$. Also by definition 1.5 [vi],

we have $H_n^* \subseteq H = ncl(H) \subseteq V$ this implies that $H_n^* \subseteq V$.

In example 2.4 and $NI_{\hat{g}\alpha}cl(U, I, N) = \{U, \emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b\}, \{b, c\}, \{a, c, d\}, \{a, b, c\}, \{b, c, d\}\}$.

The set $\{b, d\}$ is $\bar{\rho}gI$ -closed but not $NI_{\hat{g}\alpha}$ -closed.

Theorem 2.9 Every $NI_{g\alpha}$ -closed set is $\bar{\rho}gI$ -closed but not conversely.

Proof: Given that H is $NI_{g\alpha}$ -closed set we have $N\alpha cl(H) \subseteq V$ where $H \subseteq V$,

V is nano open set, By theorem 1.6(3), V is NIg semi*-open set containing H . Also $H_n^* = ncl(H)$

$\Rightarrow H_n^* \subseteq N\alpha cl(H) \subseteq V$

$\Rightarrow H_n^* \subseteq V$.

Thus H is $\bar{\rho}gI$ -closed.

Example: In example 2.7, $NI_{g\alpha}$ -closed set = $\{U, \phi, \{a\}, \{b\}, \{a,c\}, \{a,b\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c\}, \{b,c,d\}\}$.

The set $\{c\}$ is $\bar{\rho}gI$ -closed but it is not $NI_{g\alpha}$ -closed set.

Theorem 2.10 Every Nano I-closed set is $\bar{\rho}gI$ -closed but not conversely.

Proof: Let H be Nano I-closed and V be NIg semi*-open in H .

By theorem 1.6(1), $H_n^* = ncl(H_n^*) = ncl(H)$

Take $H \subseteq V$, $H_n^* = ncl(H_n^*) \subseteq H$.

$H_n^* \subseteq H \subseteq V$. Thus H is $\bar{\rho}gI$ -closed.

In example 2.7 & NI -closed set = $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b\}, \{a,d\}, \{b,c\}, \{a,b,c\}\}$.

The set $\{a,c,d\}$ is $\bar{\rho}gI$ -closed but not NI -closed.

Theorem 2.11 If H is ng closed iff H is $\bar{\rho}gI$ -closed. Where $I = \{\phi\}$ in (U, I, N) .

Proof: Let $I = \{\phi\}$, H be ng -closed. $\Leftrightarrow H_n^* \subseteq H \subset ncl(H)$ also every subset of V is n^* -dense in itself. Hence proved.

Theorem 2.12 Every Nano closed set is $\bar{\rho}gI$ -closed but not conversely.

Proof: Let H be nano-closed set and $H \subseteq V$. where V is NIg semi*-open in H . Since H is nano -closed we have $ncl(H) = H$. But $H_n^* \subseteq ncl(H) = H \subseteq V$

$\Rightarrow H_n^* \subseteq V$. Hence proved.

Example 2.13 Let $U = \{a,b,c,d\}$, $X = \{a,d\} \subset U$ and

$\tau_R(X) = \{U, \phi, \{d\}, \{a,c\}, \{a,c,d\}\}$,

$\bar{\rho}gI$ -closed set = $\{U, \phi, \{b\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$.

Clearly the set $\{a,b,d\}$ is $\bar{\rho}gI$ -closed but not Nano-closed.

Theorem 2.14 A subset H of U in a space (U, I, N) is $\bar{\rho}gI$ -closed iff $ncl^*(H) \subseteq V$ whenever $H \subseteq V$ and V is NIg semi*-open.

Proof: Let $H \subseteq V$ where V is NIg semi*-open.

Since H is $\bar{\rho}gI$ -closed we have $H_n^* \subseteq V$. Thus $H \subseteq V$ and $H_n^* \subseteq V$

$\Rightarrow ncl(H_n^*) = H \cup H_n^* \subseteq V$. This proves necessary part.

Conversely, Suppose $H \subseteq V$ where V is NIg semi*-open by assumption,

$ncl^*(H) \subseteq V \Rightarrow H \cup H_n^* \subseteq V \Rightarrow H_n^* \subseteq V$. Hence H is $\bar{\rho}gI$ -closed.

Remark: The intersection of two $\bar{\rho}gI$ -closed set is need not be $\bar{\rho}gI$ -closed as seen in the following:

Example 2.15 Let $U = \{a,b,c,d\}$, $I = \{\phi, \{c\}\}$ and

$\bar{\rho}gI$ -closed set = $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$.

Clearly the set $\{a,c,d\} \cap \{b,c,d\} = \{c,d\}$ is not in $\bar{\rho}gI$ -closed.

Theorem 2.16 If H and K are $\bar{\rho}gI$ -closed set then $H \cup K$ is also $\bar{\rho}gI$ -closed.

Proof. Given H and K are $\bar{\rho}gI$ -closed set. Then $H_n^* \subseteq V$ where $H \subseteq V$ and V is NIg semi*-open and $K_n^* \subseteq V$ where $K \subseteq V$ and V is NIg semi*-open. Since H and K are subsets of V , $(H_n^* \cup K_n^*) = (H \cup K)_n^*$ is a subset of V and V is NIg semi*-open. Hence $H \cup K$ is $\bar{\rho}gI$ -closed.

Theorem 2.17 If $H \subseteq X$, then H is $\bar{\rho}gI$ -closed iff $n-cl^*(H) \subseteq V$ whenever $H \subseteq V$ and V is NIg semi*-open in V .

Proof : Assume H is a $\bar{\rho}gI$ -closed, we have $H_n^* \subseteq V$ where $H \subseteq V$ and V is NIg semi*-open in U . Now $n-cl^*(H) = (H \cup H_n^*) \subseteq V$ whenever $H \subseteq V$ and V is NIg semi*-open.

Conversely, Let $H \subseteq V$ and V be NIg semi*-open in U . By hypothesis $n-cl^*(H) \subseteq V$.

Since $n-cl^*(H) = (H \cup H_n^*)$, we have $H_n^* \subseteq V$.

Theorem 2.18 If (U, I, N) is any ideal nano topological space and $H \subseteq U$, then the following are equivalent.

1. H is $\bar{\rho}gI$ -closed
2. $n-cl^*(H) \subseteq V$ whenever $H \subseteq V$ and V is NIg semi*-open in U
3. For all $h \in n-cl^*(H)$, NIg semi* $cl(\{h\}) \cap H \neq \phi$
4. $n-cl^*(H) - H$ contains no nonempty NIg semi*-closed set
5. $H_n^* - H$ contains no nonempty NIg semi*-closed set

Proof: (1) \Rightarrow (2) If H is $\bar{\rho}gI$ -closed, then $H_n^* \subseteq V$ whenever $H \subseteq V$ and

V is NIg semi*-open in U and so $n-cl^*(H) = H \cup H_n^* \subseteq V$ whenever $H \subseteq V$ and

V is NIg semi*-open in U .

(2) \Rightarrow (3) Suppose $h \in n-cl^*(H)$. If NIg semi* $cl(\{h\}) \cap H = \phi$,

then $H \subseteq U - NIg$ semi* $cl(\{h\})$. By (2), $n-cl^*(\{h\}) \subseteq U - NIg$ semi* $cl(\{h\})$ and

hence $n-cl^*(H) \cap \{h\} = \phi$, a contradiction, since $h \in n-cl^*(H)$.

(3) \Rightarrow (4) Suppose $F \subseteq n\text{-cl}^*(H) - H$, H is Nlg semi*-closed and $h \in H$.

Since $H \subseteq U - H$, $F \cap H = \emptyset$. We have $\text{Nlg semi}^* \text{cl}(\{h\}) \cap H = \emptyset$

because H is Nlg semi*-closed and $h \in H$. From (3), which is a contradiction. Therefore $n\text{-cl}^*(H) - H$ contains no nonempty Nlg semi*-closed set.

(4) \Rightarrow (5) Since $n\text{-cl}^*(H) - H = (H \cup H_n^*) - H = (H \cup H_n^*) \cap H^c = (H \cap H^c) \cup (H_n^* \cap H^c) = H_n^* - H$. Therefore $H_n^* - H$ contains no nonempty Nlg semi*-closed set.

(5) \Rightarrow (1) Let $H \subseteq V$ where V is Nlg semi*-open set. Therefore $U - V \subseteq U - H$

and so $H_n^* \cap (U - V) \subseteq H_n^* \cap (U - H) = H_n^* \cap H^c = H_n^* - H$.

Therefore $H_n^* \cap (U - V) \subseteq H_n^* - H$. Since H_n^* is always n -closed set,

so H_n^* is Nlg semi*-closed and so $H_n^* \cap (U - V)$ is a Nlg semi*-closed set

contained in $H_n^* - H$. Therefore $H_n^* \cap (U - V) = \emptyset$ and hence $H_n^* \subseteq V$.

Therefore H is Nlg semi*-closed.

Theorem 2.19 Let $H \subseteq U$ is an $\bar{\rho}gI$ - closed set then the followings are hold

1. H is an n^* -closed set.

2. $n\text{-cl}^*(H) - H$ is Nlg semi*-closed set.

3. $H_n^* - H$ is Nlg semi*-closed set.

Proof: (1) \Rightarrow (2) If H is n^* -closed, then $H_n^* \subseteq H$ and so $n\text{-cl}^*(H) - H = \emptyset$.

Therefore $n\text{-cl}^*(H) - H$ is Nlg semi*-closed.

(2) \Rightarrow (3) since $n\text{-cl}^*(H) - H = H_n^* - H$, it is clear.

(3) \Rightarrow (1) If $H_n^* - H$ is Nlg semi*-closed set and H is Nlg semi*-closed set, by theorem [3.18] $H_n^* - H = \emptyset$ and so H is n^* -closed.

Theorem 2.20 Every subset of U is $\bar{\rho}gI$ - closed set iff every Nlg semi*-open set is n^* - closed.

Proof: suppose every subset of U is $\bar{\rho}gI$ - closed. If $M \subseteq U$, U is Nlg semi*-open then M is $\bar{\rho}gI$ - closed and $M_n^* \subseteq M$. Hence M is n^* -closed.

Conversely, Suppose that every Nlg semi*-open set is n^* -closed. If M is Nlg semi*-open set such that $H \subseteq M \subseteq U$ then $H_n^* \subseteq M_n^* \subseteq M$ and so H is $\bar{\rho}gI$ - closed.

Theorem 2.21 Let (U, I, N) be an ideal nano topological space and $H \subseteq S$.

Then H is $\bar{\rho}gI$ - closed iff $H = F - M$ where F is n^* -closed and M contains no nonempty Nlg semi* closed set.

Proof: If H is $\bar{\rho}gI$ - closed, then $M = H_n^* - H$ contains no nonempty Nlg semi*-closed set. If $F = n\text{-cl}^*(H)$, then F is n^* -closed such that

$$F - M = (H \cup H_n^*) - (H_n^* - H)$$

$$= (H \cup H_n^*) \cap (H_n^* \cap H^c)^c = (H \cup H_n^*) \cap ((H_n^*)^c \cup H)$$

$$= H \cup ((H_n^* \cap H^c)^c \cap (H_n^*)^c) = H.$$

Conversely, suppose $H = F - M$ where F is n^* -closed and M contains no nonempty Nlg semi*-closed set. Let V be Nlg semi*-open set such that $H \subseteq V$.

Then $F - M \subseteq V \Rightarrow F \cap (U - V) \subseteq M$. Now $H \subseteq F$ and $F_n^* \subseteq F$ then $H_n^* \subseteq F_n^*$ and so

$H_n^* \cap (U - V) \subseteq F_n^* \cap (U - V) \subseteq F \cap (U - V) \subseteq M$. since $H_n^* \cap (U - V)$ is Nlg semi*-closed, $H_n^* \cap (U - V) = \emptyset$ so that $H_n^* \subseteq V$. Hence H is $\bar{\rho}gI$ - closed.

3. On $\bar{\rho}gI$ – Open set in Ideal Nano Topological Space

Definition 3.1

A subset H of an ideal nano topological space is called $\bar{\rho}gI$ – Open if its complement of $\bar{\rho}gI$ – closed set in (U, I, N) .

Theorem 3.2 Let (U, I, N) be an Ideal nano topological space then following statements are hold

i) Every Nano- open set is $\bar{\rho}gI$ – Open

ii) Every NanoI- open set is $\bar{\rho}gI$ – Open

iii) Every n^* open set is $\bar{\rho}gI$ – Open

iv) Every N_g open set is $\bar{\rho}gI$ – Open

v) Every $NI_{\bar{g}}$ open set is $\bar{\rho}gI$ – Open

vi) Every $NI_{\bar{g}\alpha}$ open set is $\bar{\rho}gI$ – Open

vii) Every $NI_{g\alpha}$ open set is $\bar{\rho}gI$ – Open

Proof:

i) Let H be a nano-open then H^c is nano-closed.

By theorem[2.12], H^c is $\bar{\rho}gI$ – closed. Thus H is $\bar{\rho}gI$ – open.

ii) Given that H be a nanoI-open then H^c is nanoI-closed.

By theorem[2.10], H^c is $\bar{\rho}gI$ – closed. Thus H is $\bar{\rho}gI$ – open.

iii) Let H be a n^* -open, then H^c is n^* -closed.

By theorem[2.3], H^c is $\bar{\rho}gI$ – closed. Thus H is $\bar{\rho}gI$ – open.

iv) Assume that H is N_g -open. Then H^c is N_g -closed.

By theorem[2.5], H is $\bar{\rho}gI$ – open.

v) Let H be a $NI_{\bar{g}}$ open set. Then H^c is $NI_{\bar{g}}$ -closed.

By theorem[2.6], H is $\bar{\rho}gI$ – open.

vi) Given that H is $NI_{\bar{g}\alpha}$ open. Then H^c is $NI_{\bar{g}\alpha}$ -closed.

By theorem[2.8], H^c is $\bar{\rho}gI$ – closed. Thus H is $\bar{\rho}gI$ – open.

vii) Let H be $NI_{g\alpha}$ open. Then H^c is $NI_{g\alpha}$ -closed.

By theorem[2.9], H^c is $\bar{\rho}gI$ – closed. Hence H is $\bar{\rho}gI$ – open.

Remark : The converse of the above theorem is need not be true as shown in the examples [2.2, 2.4, 2.7, 2.13]

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